Ergodesign & HCI
número 1, volume 4, ano 4 (2016)
ISSN 2317-8876, Rio de Janeiro - Brasil

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Augmented Reality: Towards a visual and tangible learning of calculus

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Augmented Reality, Visualization, Educational Technology

In order to take advantage of the didactic potential that Augmented Reality (AR) provides we present an educational resource meant to help to transform the teaching and learning of Mathematics, through the creation of graphical representations for mathematical reasoning. The spatial visualization skill is a cross-curriculum content that has been taken for granted, the challenge then is to improve its development. With the design of this AR application we want to help students with this task. The application covers content that belongs to conventional courses of calculus I, II and III at college.

1. Introduction

An emergent technology such as Augmented Reality (AR) provides the opportunity to change the way in which students interact with Mathematics. The mathematical knowledge involves dealing with symbolic representations necessarily. Numerical, algebraic and graphic representations are the standard, and it is not just important to perform tasks on each of them, the exchange between those representations is an essential skill for the learning of mathematics (Duval 2006).
Based on previous inquiry we consider that the graphical representation is the one that requires a greater cognitive effort in order to understand and explain a mathematical behavior. With that conviction, our research agenda has been focusing in the design of visualization proposals for the students in order to foster their interaction with mathematical knowledge in an innovative way. We believe that a well-conceived graphical representation can provide the students' mind a steady support in order to deal with the other mathematical representations.
Our viewpoint is that the problems related to the learning of mathematics are associated specifically in how we conceive this science and how it is reflected in our teaching process. Therefore, we have been working in a new pedagogy developed to approach the students with the revised work of Alanís & Salinas (2010), Salinas & Alanís (2009), Salinas et al. (2011) in order to change the perception of students about the learning of mathematical knowledge. The graphical representation is enhanced by the affordances, in the sense of Norman (1999), that are provided by the AR visualization design of the mathematical content.
A strong standpoint in reference with the use of digital technology in mathematical education is the opportunity to create new forms of symbol-focused experiences. We strongly search for an understanding in which the digital technologies can effectively be integrated into mainstream education and become an active participant of the cognitive

process as referred by Moreno-Armella & Hegedus (2009), Moreno-Armella & Sriraman (2005). The concept of co-action especially guides the design process in the development of the AR application, seeking to cope with the cognitive efforts of the student and the visualization provided by the Graphic User Interface (GUI) design.
This inquiry standpoint emerges as the result of the integration of technology in the learning process within the educational model of our institution, the Monterrey Institute of Technology and Higher Education in Mexico. The particular concern of our research group is in the Mathematics Education, as part of an innovation program where the use of emergent technologies and educational strategies nourish a positive attitude from the students when learning mathematics.
Nowadays, the research group is circumscribed in the challenges presented by the multidisciplinary tasks that are required for leading us to this innovation. Technology changes the way people learn, and a constant and rapidly transformation of how knowledge is delivered, requires the conjunction of academics, designers and software developers working together. Beside the technical challenges and considerations to create and produce a technological product, it is necessary to stress that even more important than the technology innovation per se, is the new teaching strategies in which students and teachers can focus their attention as active participants in the learning process.

2. Augmented Reality App

2.1 Why an AR App?

The AR App is an approach to reach and engage students habituated to use mobile devices in their daily life. The opportunity relies in the chance to focus their attention in a usual task for them, the use of certain device to get some sort of information, in our case, the learning of calculus courses. The presence of technology, in different aspects of life, creates the necessity to develop certain skills and competences in order to learn at fast rate because technology is constantly changing. The users must learn to learn and keep their digital skills updated. According to the Program for International Student Assessment (PISA), teachers are using educative practices from the XX century using technology from the XXI century. Therefore, new educational practices must be developed to seize the advantages provided by certain technologies as tools. Teachers must consider the learning environments and the technological mediums as well as instructional design. A great technology can enhance a great education; nevertheless, a great technology can't replace a poor education (Shapiro, 2015).
Bennett, Bishop, Dalgarno, Waycott and Kennedy (2012) mention that mobile technology is increasingly popular in the daily lives of students, so it is necessary that teachers begin exploring the formal use of them in education. These authors add that the potential is in the mode of involvement with students and the materials made by using technology. They also add that the use of these tools is a required competence in the contemporary world.
By knowing the traditional curriculum of calculus, and looking for a new curriculum assisted by technology, we identified spatial visualization. This is a mathematical skill useful for the first 3 mathematics courses at college, so it deserves to be explored with an AR application. The AR App should be available for mobile devices, as previously stated, because of the availability in the student's hands and because the high chance to deal with technology literate users.

2.2 The AR Application: Towards a visual and tangible math

The development of the application required a multidisciplinary group composed by academics specialized in the teaching of Mathematics, interaction designers and software developers. Previously to the layout and mock up of the screens in the application, a curricular revision was necessary to identify which contents would be covered and available in the App.
After a review of the mathematical content that would be used, the design and software development process began. The interface design was constructed from the initial sketches to the mock-ups that the developers used to scaffold the GUI in the SDK named Unity. In parallel to this process, the 3D elements were modelled in 3D authoring tools such as Maya and Rhino and then they were also imported to the Unity motor to generate the packages required for Android and iOS. A second step was needed to compile for iOS from the Xcode platform. In this way, the app would be available for two different mobile operative systems increasing the scope of devices that could be used in the classroom.
The final structure of the application consisted in three sections named: *From 2d to 3D, Solids of Revolution* and *3D Surfaces,* which can be associated with Mathematics I, II and III. The

navigation on the app works based on traditional UI conventions such as a side panel for the controls, different buttons that controls the actions in the display in relationship with the content displayed, a window for video explanation of what is happening in the 3D animation, and also a window for the mathematical expression to be displayed.

2.2.1 From 2D to 3D

This is the initial part of the AR app and studies the transition from a 2D curve to a 3D surface, through “accumulation in time-space” of different curves. It starts with an initial known curve-form where different graphical effects take place as shown in Table 1. The curve will be elaborated by the graphical effect that corresponds to the presence of the parameter k in the algebraic expression.

The curve images are in 2D originally; the animation is triggered simulating a continuous 3D surface being created. Successive curves are placed in parallel planes situated closely in a way that the surface begins to create its own shape. With the effect of the parameter (k) and simultaneously the “motion of time” through the successive copies in parallel planes, the 3D visualization takes place in real place and time. Figure 1 exemplifies the animation, inviting to think in the accumulation of 2D curves, in space and time, creating a 3D surface.

2.2.2 Solids of revolution

Moving on with the second section, the application considers four curves behavior, as shown in table 2. A total of 24 simulations compose this section. The menus and UI elements stay consistent with the previous sections, but there are some options that

Table 1. Different graphical simulations for section 1

Graphical effect	Parabola	Sine	Circle
Original	$y = x^2$	$y = \sin x$	$y = \sqrt{1 - x^2}$
Vertical scroll	$y = x^2 + k$	$y = \sin x + k$	$y = \sqrt{1 - x^2} + k$
Horizontal scroll	$y = (x + k)^2$	$y = \sin(x + k)$	$y = \sqrt{1 - (x + k)^2}$
Contraction Expansion	$y = kx^2$	$y = k \sin x$	$y = k\sqrt{1 - x^2}$
Other		$y = \sin kx$	$y = \sqrt{R^2 - x^2}$

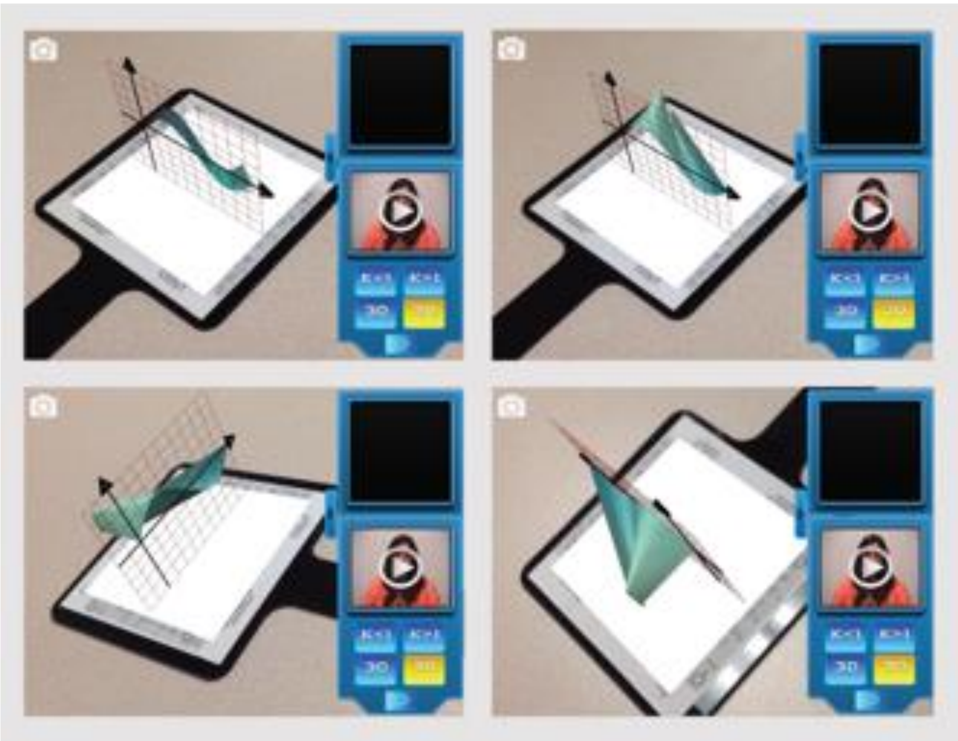

Fig. 1 Frames from a simulation included in the first section: from 2D to 3D

change depending on the content that is going to be displayed.

In this section, each curve behavior includes both, the case in which the rotation of the curve is performed around the x-axis, and also the case with rotation around the y-axis. In each case, there are 3 different ways for visualizing the creation of the solid of revolution. First, the natural simulation conceiving rotation of the area; second, the simulation suggested by the method for volume calculation by "disks", and third, the simulation suggested by the method of volume calculation by "shells".

Taking into account that spatial visualization skill should not be considered inborn, and that students manifest different levels of its development, it was decided to include first the visualization of the solid of revolution in the natural way, as rotating a bounded region. As we care for the identification of the mathematical integral expression to calculate the volume, we included the simulation of the rotating disks and shells in order to promote the visualization of the same solid but now accumulating disks or shells. Figure 2 illustrates part of the first visualization simulation, evoking a natural cognitive process to visualize it.

Table 2. Different graphical simulations for section 2

Curve Behavior	Function	Rotation	Simulation
Concave upwards-increasing	$y = x^2$	x axis	Visualizing
Concave upwards-decreasing	$y = 2 - \sqrt{x}$	y axis	Disk method
Concave downwards-increasing	$y = \sqrt{x}$		Shell method
Concave downwards-decreasing	$y = 4 - x^2$		

2.2.3 3D Surfaces

The last section of the AR app consists in visualizing 3D surfaces in a different way, now seen as the graph of a two-variable mathematical function. We selected a diversity of functions to present having different features in order to illustrate the kind of behavior that is possible to conceive in a 3D space. Table 3 shows the list of functions considered, as well as the curves of intersection with the surface that are generated by planes parallel to the XY, XZ and YZ planes.

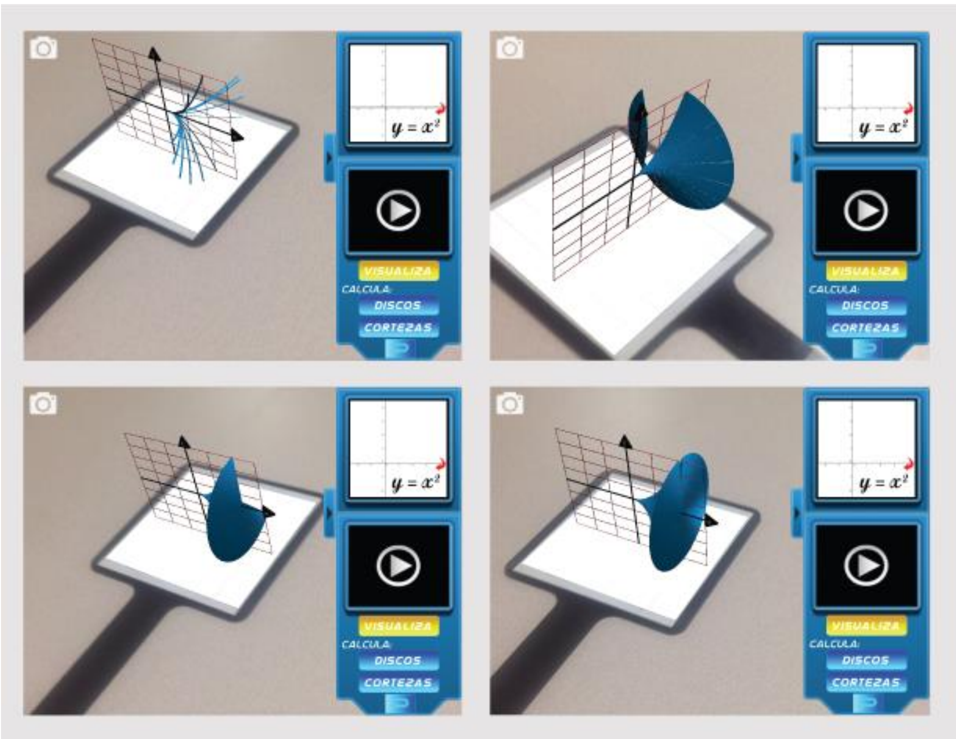

Fig. 2 Different frames from the solid of revolutions simulation in section 2

Table 3. Different graphical simulations performed in section 3

Function	Intersections	Visualization
$z = x^2 + y^2$	xz plane	3D
$z = x^2 - y^2$	yz plane	2D ↔ 3D
$z^2 = x^2 + \frac{y^2}{25}$	xy plane	
$z = \frac{1}{2}\lvert x^2 - y^2 \rvert$		
$z = y \sin x$		

The innovation introduced in this level is the simulation of a visual process but now performed from 3D to 2D. Originally there is the AR model in 3D with the surface, the graph of the function. This continues with a sequence of planes that get into and out of the scene and intersect the surface. There is a visually return to 2D when the surface in 3D is affected by the intervention of planes parallel to the coordinate planes XY, YZ and XZ.
This visual process of cutting and rebuilding the surface promotes a cognitive evocation of the new way to visualize the surface. Figure 3 shows part of the simulation in section 3.

opportunities that the emergent digital technologies can provide for the learning of mathematics. Nowadays top educational institutions around the world are taking care for the promotion of innovation in the educational environment. Redefinition in the use of time and space for teaching and learning make us aware of the great potential that emergent technologies could afford. As a research group the objective is to look for the possible use of these emergent technologies and how and why we could use them in the learning process.
For Collins and Halverson (2010) this is a second educational revolution, the first one resulted in the

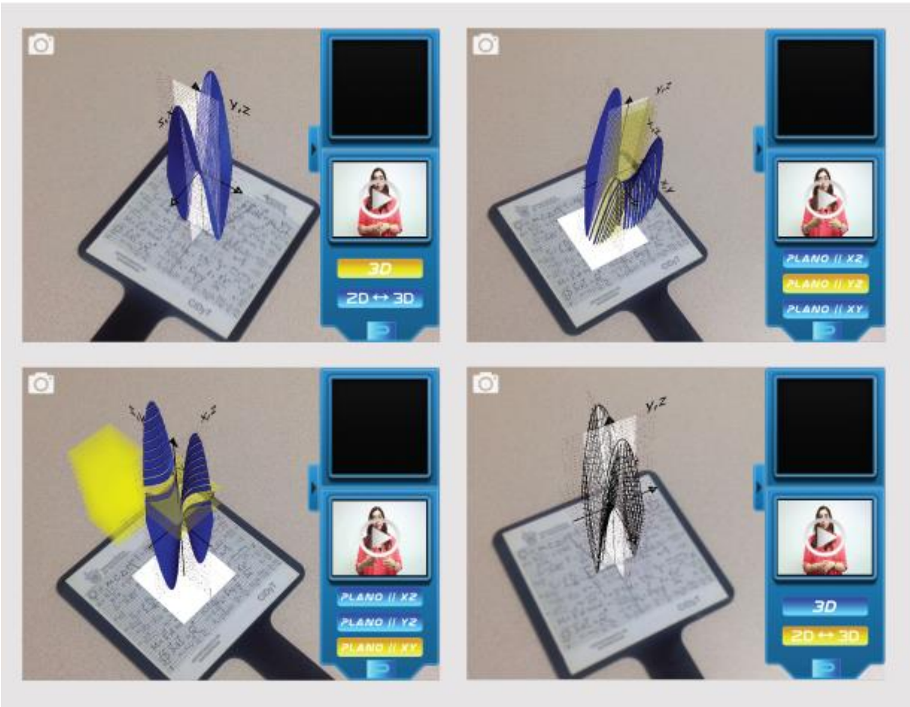

Fig. 3 Different graphical simulations performed in section 3

3. Conclusions

The research agenda of this multidisciplinary team is deliberately focused to cope with the new

Educational Systems of the Industrial Revolution. This second revolution occurs due to the use of digital technology, such as computers, mobile devices, digital content distribution tools, video games and social networks. In this context the learner has the possibility to adapt their learning to their own terms by using technology, so it can develop learning activities without physical, time or content restrictions. Users can choose to learn at the time that suits them and what most interested them. We take part in this redefinition and its impact for the learning of mathematics with the integration of new digital technologies for developing mathematical skills. With this application we intend to offer students a visual and tangible mathematics, where mathematical representations could be perceived in a real place and time. They can interact with the App inside and outside the classroom; it is like a personal ally that gives them the opportunity to interact with their own thinking process. It has been our main concern to conceive an AR App that provides a simulation to be easily adopted cognitively. Spatial visualization is not a mathematical topic considered in curriculum, and we are not saying it should be, but is quite a mathematical skill useful for mathematics learning. What we seek is to support formal education with digital contents that make mathematics knowledge affordable.

As a research team we continue analyzing and identifying mathematical skills in curriculum that make a great difference when learning mathematics, and trying to integrate digital technology in order to develop those skills. It is our conviction that this way, mathematical knowledge should be better understood for students nowadays, and it can lead them to have a better attitude for its learning.

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